Assessing Service Operations in the Short and Long Runs: efficiency analysis and empirical application

FREDERICO A. DE CARVALHO
UFRJ
fdecarv@msn.com

MARCELINO JOSÉ JORGE
Universidade Federal do Rio de Janeiro
marcelino.jorge@ipec.fiocruz.br

ÁLVARO VIEIRA LIMA
Universidade Estadual do Rio de Janeiro
alvarolima@attglobal.net
1 Introduction

In this paper we present an optimization approach to assessing the performance of service organizations from an Efficiency Analysis standpoint. In what concerns the empirical illustration, even though the paper employs data from a sample of Decision Making Units (DMUs) pertaining to a public system of academic libraries, there is no loss of generality if and when other kinds of organizations are considered. Summing up, our approach combines in a simple way efficiency scores computed from the estimation of selected Data Envelopment Analysis (DEA) models and a long run evaluation provided by Markovian analysis.

The text is organized in five sections that include this introduction. The second section brings together some background ideas and results that helped found the paper. In the next section the methodological procedure is explained, followed by empirical findings in the fourth section. Conclusions, limitations and pending issues are presented to close the text.

2 Background

The proposed approach relies essentially on the application of the so-called Efficiency Principle to assess organizational performance in a public university system of libraries. Following the literature, “organization” may be taken quite broadly as meaning both public and private entities, and even nonprofit ones among the latter (Nunamaker, 1985; Fox, 2001; Vakkuri, 2003; Smith and Street, 2005; Afonso, Schuknecht and Tanzi, 2006,).

Data Envelopment Analysis (DEA)

The Efficiency Principle simply states that, when studying the production process in any organization, whenever a production unit uses the same resources but yields greater quantities of output than another unit, it should be considered “relatively more efficient” (i.e., relative to one another), no matter how formally the productivity problem is analyzed. Analogously it should be considered “relatively more efficient” if it uses fewer resources and yields the same output. From an analytical standpoint these properties correspond to evaluating a library unit in terms of its position 

\[ \text{vis à vis} \]

an adequately defined and computed “efficiency frontier”, that is, the locus of all “equally best productive combinations of inputs and outputs”. Once identified the frontier, the performance of a specific library system may be evaluated by assessing the relative position of its component units relatively to each other and to the frontier.

Although it may seem restrictive to anyone aiming at “comprehensively tackling” the complexities of organizational assessment, no doubt the Efficiency Principle states an idea that very few would agree to dispose of. In addition there is an established body of knowledge – namely, Data Envelopment Analysis (DEA), a class of mathematical programming models – with a now long tradition (Emrouznejad, Parker and Tavares, 2008) of being applied to a broad range of situations involving the analysis of production frontiers in a multi-unit, multi-input and multi-output framework in such a way that usual parametric restrictions are absent. The so called nonparametric models of frontier adjustment, such as DEA, represent the efficiency frontier as the best observed practices, that is, as the maximum output obtained
from an input bundle when considering all the empirically observed organizational units in the population studied.

In applied work DEA has been used to evaluate several types of organizations, such as libraries (Chen, 1997-a, -b; Stancheva & Angelova, 2004; Midda & Kikas, 2009), industrial plants, bank branches, education systems, hospitals and military units or systems, all properly understood as types of "complex organizations" (EMROUZNEJAD, PARKER & TAVARES, 2008).

This flexible and widespread applicability stems from the fact that a DEA model does not request the predefinition of a functional form for the production function, as it is the case in econometric regression approaches, also long employed in the case of public libraries (for instance, VITALIANO, 1997, 1998).

Among the characteristics of interest of the DEA model that are relevant for the assessment of public organizations - subject to operate under the restriction of a budget _a priori_ limited – mention must be made of the possibility to include in the analysis several inputs and outputs estimated by different units of measurement. It is also worth mentioning that the direct use of any empirically available inputs and outputs eliminates the need to define or redefine either resource or performance “indicators” of any type such as can be frequently found in the literature.

**Efficiency Analysis in the long run**

In a seminal methodological paper Tulkens and Vanden Eeckaut (1995) describe and explain the main issues relating to the role of time in nonparametric efficiency analysis, especially in what concerns alternative ways to accommodate empirical information into reference production sets that will be submitted to efficiency computations. Although they do not explicitly mention the long run, their presentation suffices to establish a neat picture on each possible approach to panel data efficiency analysis. Of particular interest here (see Table 1) is their classification (Ibid., p. 478-480) of the variety whereby the time dimension present in panels may be treated when investigating observed productive activity.

Semenick Alam and Sickles (2000), Ahn, Good and Sickles (2000), and Wang and Huang (2007) are interested in directly tackling the long run into (in)efficiency analysis. The first two papers do this essentially by econometric techniques apt to specify a lag structure for the estimation of models of panel data in such a way that (long run) equilibrium may be discussed with appropriate techniques related to solving difference equations (e. g. cointegration analysis in nonparametric applications; see also the pioneering Sengupta, 1992).

The paper by Wang and Huang (2007) introduces two new models to examine long run efficiency analysis:

(a) a dynamic panel data model with a lagged dependent variable that happens to be endogenous with respect to both errors and the intercept in the equation to be estimated – so that conventional estimators are not enough and are conveniently replaced (p. 1306); this model allows to estimate the size of dynamic adjustment costs; and

(b) a two-state Markov Chain model leading to the estimation, for each DMU, of its efficiency status as specified in their equation (2.12) (p. 1307).

According to the authors “the Markov model is mainly designed to uncover a potential link between financial indicators and the flow between states” (p. 1307) and provides “valuable
information (…) which renders opportunities to regulators and managers reallocating scarce supervisory resources” (ibid.). Specifically the Wang-Huang Markov model allows discussing long run evolution of the efficient status in two ways:

(i) first, for each DMU, in terms of equilibrium values of efficient status by dealing with the corresponding difference equation (2.12) (p. 1307);

(ii) second, for the set of DMUs, in aggregate (or “structural”, see Sengupta, 1997) terms by considering the difference equation in (2.5) (p. 1306).

However they do not further pursue these ideas and do not compute any long run solutions in any of those cases. In addition, although they have modeled and specified the probability of one-step temporal transition from efficient (resp. inefficient) to inefficient (resp. efficient) state, there seems to be no indications as to how those probabilities might be used to compute long run “structural” distributions of the DMUs among the two states (“efficient” or “inefficient”).

Using results from finite ergodic Markov chains (Kemeny and Snell, 1972, p. 130-131), and assuming one (estimated) aggregate transition matrix is available, it is possible to compute the long run distribution of the “system” (the set of DMUs) between the two states. This is an important goal to be pursued in this paper.

3 Method

Our proposed assessment procedure consists of three steps. The first two steps – involving the computation of efficiency scores and of operational plans in turn – are typically performed in many applications of Data Envelopment Analysis to empirical data on DMU performance. The third, a novel one, incorporates the “structural” long run assessment of efficiency.

Data collection

The case is summarized in Table 1 and follows the Tulkens and Vanden Eeckaut (1995) framework. We focus on Brazilian data collected from an integrated system of academic libraries pertaining to a traditional federal university in Rio de Janeiro.

Our example is supported by a convenience sample of 37 library units that correspond to more than 80% of the total population and that were selected for ease of access and overall data availability. Time periods refer to 2000 – 2007. Data were collected from the libraries’ centralized MIS and relate to three inputs (number of employees, physical area in square meters and number of volumes) and four outputs (number of visits, of loans, of registers and of consultations).

Efficiency Analysis

The efficiency of productive units has been calculated by means of a deterministic production frontier, whose construction process is implemented as the solution of a linear programming problem. This procedure, known as Data Envelopment Analysis (DEA), was initially
introduced in the literature by Charnes, Cooper and Rhodes (1978, 1981) and later modified by Banker, Charnes and Cooper (1984). The most important difference between those two models is the possibility of tackling scale economies. The Banker, Charnes and Cooper model (BCC model), used in this study, allows to calculate a deterministic production frontier with variable scale yields. In addition, given the a priori restricted nature of public budgets, the output-oriented version was adopted. In this version the optimization problem to be solved is an output maximization problem such as

\[
\text{Max}_{\mu, v} \left( \mu' y / v' x_i \right), \quad \text{subject to:}
\]

\[
\mu' y / v' x_i = 1,
\]

\[
\mu' y / v' x_j \leq 0, \quad j=1, 2, \ldots, N,
\]

\[
\mu, v \geq 0. \quad (3.1)
\]

The solution of the appropriate linear programming problem provides numerical scores for each DMU that characterize them with respect to efficiency status. For each inefficient DMU an operation plan is also provided that indicates (re)allocative targets for the DMU to reach efficiency. Finally scores will also be needed to compute the transitions between the two states along the time period for the whole set of libraries.

**Markovian Analysis**

As soon as a transition matrix is available, first passage time and long run analysis are possible and will result from the computation of a fixed point for the transition matrix (Kemeny and Snell, 1972, p. 130-131). This fixed point is a probability vector containing the distribution of the “system” between the two states in the long run.

In order to get a transition matrix from empirical data about the temporal behavior of a “system” of states it suffices to use the transition count (Anderson and Goodman, 1957; Billingsley, 1961; Wang and Huang, 2007, p. 1306) corresponding to the proportion of units in a given state and then count the transition between each pair of states in the period.

In the present application there are six observed transition matrices, one for each successive pair out of seven years. If only the first transition matrix would be used to compute the fixed point, no doubt much information would be ignored. Since we do not follow the econometric approach (e. g., Wang and Huang, 2007), some other choice should be made.

According to Kemeny and Snell (1972, p.131), when the number of time steps grows indefinitely one has

\[
\lim (1/n)(P + P^2 + \ldots + P^n) = [1 \ 1 \ 1 \ 1]' \pi \quad (3.2),
\]

where \( n \) is the number of steps; \( P^n = (( p_{ij}^{(n)} )) \) is the nth power matrix of the one-step transition matrix \( P \), whose \(( i ; j )\) element then represents the probability of transition from state \( i \) to state \( j \) after \( n \) steps; \([1 \ 1 \ldots, 1]' \) is a column vector with all elements equal to 1, and \( \pi \) is precisely the fixed point, that is, a constant vector containing the long run equilibrium distribution between states whose components are nonnegative and sum to 1 (as any probability vector), and such that \( \pi P = \pi \).

Note that the matrix product in the right hand side of (3.2) is a square matrix of the same order as \( P \) and with all lines equal to \( \pi \). The expression “long run equilibrium” is then adequate since \( \pi \) does not depend neither on time, nor on the initial state.
Also note that
\[
\lim \left( \frac{1}{n}(P^n + P^{n+1} + \ldots + P^{2n}) \right) = \lim \left[ \frac{1}{n}(P + P^2 + \ldots + P^n)P^n \right] = \\
\lim \left( \frac{1}{n}(P + P^2 + \ldots + P^n) \right) \lim P^n = [1 1 1 1]' \pi \lim P^n = \\
\lim ( [1 1 1 1]' \pi P^n ) = [1 1 1 1]' \pi. \quad (3.3)
\]

Therefore, in order to compute the fixed point \( \pi \), any power of the one-step transition matrix could be used. This is just another way to say that, since \( P^n \) is the transition matrix after \( n \) steps, the long run may be taken as starting from it as well, for any \( n \), in accordance to the intuitive notion of long run.

The first application of Markovian Analysis can be performed by using the so-called fundamental matrix (Kemeny et al., 1964, p. 405) associated to \( P^n \), the transition matrix after \( n \) steps, and its equilibrium vector to compute mean first passage time and mean recurrence time (Ibid., p. 411-414) for any state of the system.

A possible link between the mean first passage time (from a given state to another, for example, “inefficient to efficient”) and efficiency analysis stems from the fact that the time before the (mean) first passage into efficiency may suggest how urgent may be the changes indicated in the “operations plans” provided by efficiency analysis (corresponding to the second step in the proposed procedure). Analogously, the time before the first passage into inefficiency might signal to how alert managers must remain even when their initial (or present) position may be comfortable.

The second application of Markovian Analysis is also related to the fixed point of \( P^n \), since \( \pi \) directly provides the long run equilibrium of the system (Kemeny and Snell, 1972, p. 131). This equilibrium may be interpreted as the long run (percent) distribution of units between states, since system transitions between states are defined as counts of units’ transitions.

4 Results

In this section findings are presented relating to the selected academic libraries. Comments follow the order of proposed steps – computed efficiency scores, operation plans and long run distribution.

First step – efficiency scores are computed and libraries may be ranked accordingly

A sample profile for the 37 DMUs (see Table 1) is given in Table 2 for the last year of the period of study. Accordingly, computed the coefficients of variation imply that the libraries are quite different from one another on most attributes.

The basic results for any DEA analysis – namely, computed efficiency scores – appear in Table 3. Since every efficient DMU has a score equal to 1, the 8 libraries in that situation along 2000-2007 have been removed from Table 3, given that, by the very definition of efficiency, there is no way to improve their productive performance. These DMUs present a quite robust performance and deserve attention no matter how “benchmark” is understood.

Relatively inefficient DMUs receive a score less than 1. Note that some inefficient libraries never visited the efficient frontier and are even far away of it; in that sense they also deserve managerial attention. Note also the situation of library unit number 5 – it has been efficient
along the whole period except for one year. Why is that so? Should this situation be ascribed to measurement error or does it mean a real although negligible loss in performance? In terms of management action all these signals must likely be accompanied by an individual follow-up.

Table 2 – Sample profile for university libraries in 2007

<table>
<thead>
<tr>
<th>Variables</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Standard deviation</th>
<th>Coefficient of Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number Employees</td>
<td>1</td>
<td>33</td>
<td>8,41</td>
<td>8,06</td>
<td>95,83%</td>
</tr>
<tr>
<td>Total area ( m²)</td>
<td>37</td>
<td>6000</td>
<td>865,16</td>
<td>1400,03</td>
<td>161,82%</td>
</tr>
<tr>
<td>Volumes</td>
<td>872</td>
<td>277134</td>
<td>35228,92</td>
<td>53343,38</td>
<td>151,42%</td>
</tr>
<tr>
<td>Visits</td>
<td>108</td>
<td>137385</td>
<td>20974,68</td>
<td>33970,98</td>
<td>161,96%</td>
</tr>
<tr>
<td>Registrations</td>
<td>0</td>
<td>5603</td>
<td>1043,38</td>
<td>1115,40</td>
<td>106,90%</td>
</tr>
<tr>
<td>Loans</td>
<td>0</td>
<td>30191</td>
<td>5116,03</td>
<td>6578,68</td>
<td>128,59%</td>
</tr>
<tr>
<td>Consultations</td>
<td>0</td>
<td>66638</td>
<td>8091,62</td>
<td>12228,71</td>
<td>151,13%</td>
</tr>
<tr>
<td>Service mix (number)</td>
<td>5</td>
<td>13</td>
<td>9,54</td>
<td>1,87</td>
<td>20%</td>
</tr>
</tbody>
</table>

Table 3 – Efficiency scores* and yearly averages : 2000 – 2007

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,000</td>
<td>0,841</td>
<td>1,000</td>
<td>1,000</td>
<td>0,605</td>
<td>0,811</td>
<td>0,680</td>
<td>1,000</td>
</tr>
<tr>
<td>2</td>
<td>0,571</td>
<td>1,000</td>
<td>1,000</td>
<td>0,965</td>
<td>0,943</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
</tr>
<tr>
<td>3</td>
<td>0,305</td>
<td>0,936</td>
<td>0,845</td>
<td>0,661</td>
<td>0,542</td>
<td>0,846</td>
<td>0,775</td>
<td>0,574</td>
</tr>
<tr>
<td>4</td>
<td>0,989</td>
<td>0,960</td>
<td>0,769</td>
<td>0,783</td>
<td>0,829</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
</tr>
<tr>
<td>5</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>0,947</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
</tr>
<tr>
<td>6</td>
<td>1,000</td>
<td>0,696</td>
<td>0,742</td>
<td>0,494</td>
<td>0,584</td>
<td>0,757</td>
<td>0,548</td>
<td>0,650</td>
</tr>
<tr>
<td>7</td>
<td>1,000</td>
<td>0,731</td>
<td>0,870</td>
<td>0,452</td>
<td>0,353</td>
<td>0,127</td>
<td>0,466</td>
<td>0,624</td>
</tr>
<tr>
<td>8</td>
<td>0,941</td>
<td>1,000</td>
<td>0,471</td>
<td>0,559</td>
<td>0,782</td>
<td>0,650</td>
<td>0,626</td>
<td>1,000</td>
</tr>
<tr>
<td>9</td>
<td>0,620</td>
<td>0,895</td>
<td>0,712</td>
<td>0,974</td>
<td>0,619</td>
<td>0,740</td>
<td>1,000</td>
<td>0,679</td>
</tr>
<tr>
<td>10</td>
<td>0,528</td>
<td>0,660</td>
<td>1,000</td>
<td>0,779</td>
<td>0,727</td>
<td>1,000</td>
<td>0,847</td>
<td>0,646</td>
</tr>
<tr>
<td>11</td>
<td>0,404</td>
<td>0,590</td>
<td>0,287</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
</tr>
<tr>
<td>12</td>
<td>1,000</td>
<td>1,000</td>
<td>0,627</td>
<td>1,000</td>
<td>1,000</td>
<td>0,336</td>
<td>0,370</td>
<td>1,000</td>
</tr>
<tr>
<td>13</td>
<td>0,604</td>
<td>0,815</td>
<td>0,696</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>0,807</td>
<td>1,000</td>
</tr>
<tr>
<td>14</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>0,959</td>
<td>1,000</td>
<td>1,000</td>
<td>0,921</td>
</tr>
<tr>
<td>15</td>
<td>0,600</td>
<td>1,000</td>
<td>0,867</td>
<td>0,779</td>
<td>0,743</td>
<td>0,498</td>
<td>0,543</td>
<td>0,560</td>
</tr>
<tr>
<td>16</td>
<td>0,401</td>
<td>0,302</td>
<td>0,396</td>
<td>0,109</td>
<td>0,138</td>
<td>0,371</td>
<td>0,145</td>
<td>0,115</td>
</tr>
<tr>
<td>17</td>
<td>1,000</td>
<td>1,000</td>
<td>0,507</td>
<td>0,654</td>
<td>0,337</td>
<td>1,000</td>
<td>0,842</td>
<td>0,121</td>
</tr>
<tr>
<td>18</td>
<td>0,391</td>
<td>0,501</td>
<td>0,492</td>
<td>0,387</td>
<td>0,395</td>
<td>0,931</td>
<td>0,319</td>
<td>0,320</td>
</tr>
<tr>
<td>19</td>
<td>0,733</td>
<td>0,690</td>
<td>0,840</td>
<td>0,329</td>
<td>0,307</td>
<td>0,482</td>
<td>0,640</td>
<td>0,506</td>
</tr>
<tr>
<td>20</td>
<td>0,838</td>
<td>1,000</td>
<td>0,467</td>
<td>0,683</td>
<td>0,236</td>
<td>0,562</td>
<td>0,384</td>
<td>0,863</td>
</tr>
<tr>
<td>21</td>
<td>0,334</td>
<td>0,412</td>
<td>0,410</td>
<td>0,407</td>
<td>0,358</td>
<td>0,223</td>
<td>0,496</td>
<td>0,241</td>
</tr>
<tr>
<td>22</td>
<td>0,892</td>
<td>0,574</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>0,945</td>
</tr>
<tr>
<td>23</td>
<td>1,000</td>
<td>0,442</td>
<td>1,000</td>
<td>0,555</td>
<td>0,972</td>
<td>1,000</td>
<td>0,994</td>
<td>0,820</td>
</tr>
<tr>
<td>24</td>
<td>0,701</td>
<td>0,664</td>
<td>0,055</td>
<td>0,143</td>
<td>0,185</td>
<td>0,020</td>
<td>0,010</td>
<td>0,017</td>
</tr>
<tr>
<td>25</td>
<td>0,452</td>
<td>0,781</td>
<td>0,928</td>
<td>0,873</td>
<td>0,870</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
</tr>
<tr>
<td>26</td>
<td>0,562</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
</tr>
<tr>
<td>27</td>
<td>1,000</td>
<td>0,793</td>
<td>1,000</td>
<td>0,665</td>
<td>0,757</td>
<td>0,354</td>
<td>1,000</td>
<td>1,000</td>
</tr>
<tr>
<td>28</td>
<td>0,107</td>
<td>0,202</td>
<td>0,196</td>
<td>0,172</td>
<td>0,113</td>
<td>0,353</td>
<td>0,401</td>
<td>0,381</td>
</tr>
<tr>
<td>29</td>
<td>0,359</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
</tr>
</tbody>
</table>

Mean (n=37) | 0,7486 | 0,8077 | 0,7886 | 0,7691 | 0,7381 | 0,7993 | 0,7801 | 0,7663 |

Pct effic. | 45,96% | 48,65% | 48,65% | 48,65% | 40,54% | 51,35% | 54,05% | 51,35% |

Note. * - All libraries with scores equal to 1 for the whole period have been excluded.
Second step: operation plans indicate optimal changes for each library along the period

Operation plans are always produced as a typical result from a DEA solution. In the present application they are conveniently summarized in Table 4. In every individual matrix (not exhibited here) showing the allocative change for each (inefficient) library and each year, there are indications of resource decrease and output increase; this information is summarized in that table and deserves managerial attention. The same occurs as long as volume discards are concerned: they deserve special attention because collections may not be altered, as well as some individual titles (such as current textbooks or books of historical interest) should not be disposed of.

Table 4 – Average operation plans: 2000 - 2007

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Employees (number)</td>
<td>-1.44</td>
<td>-1.15</td>
<td>-0.76</td>
<td>-1.29</td>
<td>-0.93</td>
<td>-1.18</td>
<td>-0.61</td>
<td>-0.81</td>
</tr>
<tr>
<td>Area (m²)</td>
<td>-60.75</td>
<td>-71.04*</td>
<td>-29.85</td>
<td>-70.35</td>
<td>-48.94</td>
<td>-143.47</td>
<td>-88.05</td>
<td>-136.27</td>
</tr>
<tr>
<td>Volumes (number)</td>
<td>-3064.48</td>
<td>-3373.49</td>
<td>-1880.71</td>
<td>-4601.0</td>
<td>-6447.08</td>
<td>-651.77</td>
<td>-4720.75</td>
<td>-3153.65</td>
</tr>
</tbody>
</table>

Note * - this figure relates to a single library.

In any case, since there is evidence that inputs may be reduced alongside with output being increased, managers must keep alert and proactive as to take advantage from potential efficiency gains along time. Allocative changes such as those indicated in Table 4 (and much more so in individual worksheets) may also serve to compare recommended paths against observed actions in a yearly basis for each DMU and to that extent help evaluate individual performance.

Third step: first passages, mean recurrence and long run distribution for the system of libraries

Again, efficiency scores from Table 3 provide data to compute, for the whole system of libraries and the whole time period, several possible forms of transition matrices between two states - “efficient” and “inefficient”. How should they be combined into a single matrix to serve as the initial ingredient needed to apply the Markovian hypothesis? What form seems the most intuitively acceptable?

Given that we are working with contemporaneous reference sets (see Table 1), data for 2000-2007 allow obtaining an empirical version of the n-step transition required from (3.3) as the seven factor product of the seven observed one-step matrices, say \( P_1 P_2 \cdots P_6 P_7 \). Even though other such products, say involving less seven factors, would qualify from the theoretical viewpoint, since all of them are built from empirically observed “powers”, it seems preferable to use the most of available information from one-step transition matrices along the observed period, hence the seven factors.

Given that we are working with contemporaneous reference sets (see Table 1), data for 2000-2007 allow obtaining an empirical version of the n-step transition required from (3.3) as the seven factor product of the seven observed one-step matrices, say \( P_1 P_2 \cdots P_6 P_7 \). Even though other such products, say involving less seven factors, would qualify from the theoretical viewpoint, since all of them are built from empirically observed “powers”, it seems preferable to use the most of available information from one-step transition matrices along the observed period, hence the seven factors.
In order to compute both the fundamental matrix and the long run equilibrium distribution, as argued in the third section on the basis of equations (3.2) and (3.3), we employ successive products of yearly transition matrices, instead of powers of the same (initial or otherwise chosen) transition matrix. Note that since the product of two transition matrices is of the same nature, the interpretation of the seven factor product makes full sense.

Therefore we can take the seven factor product matrix $A$ defined as

$$A = P1P2P3...P6P7$$

(3.4)

as a good candidate to be used when solving the fixed point problem, since it incorporates more information than each individual matrix and any other products, in addition to being a good picture of the successive one-step, two-step until seven-step transitions, in the spirit of equation (3.2). The “seven factor product” approach simply amounts to envisage the long run as starting from the transitions occurring from the seventh year on. This more natural choice improves upon the “averaging” approach appearing in Carvalho et al. (2012-a).

Since there are only two states, it is very simple to compute the fundamental matrix. Therefore, according to Kemeny et al. (1964, p. 411), the mean first passage time from “inefficiency” to “efficiency” is approximately equal to 1 year and 10 months. This means that if a given unit is inefficient today and if no managerial action is taken, then on average it will take 22 months for the unit to become efficient. This delay may be compared to the time required for any possible remedial measures to become effective, say revised budgeting or training.

Considering (3.4), to obtain the (estimated) long run distribution of the system between the two states the fixed point equation $\pi A = \pi$ is solved to give:

$$\pi_E \text{ (percent efficient )} = 51.5\%; \quad \pi_{NE} \text{ (percent inefficient )} = 48.5\% .$$

Note that the first percent differs from the mean percent (48.7%, equal to the median) in the last line of Table 3 and these mean and median are closer to the inefficient percent. In this sense it might be argued that short run averaging is less benevolent and that long run analysis seems to be of a different nature vis-à-vis the arithmetic of numerical individual scores. Remember that products of transition matrices bring into play all the transitory visits to the two states along the time span.

The fixed point $\pi$ in the equation $\pi A = \pi$ also provides directly the mean recurrence time (Kemeny et al., 1964, p. 413) for the states of the system, that is, the mean time required before the system returns to a given state having started in that same state. The mean recurrence time is approximately equal to 2 years in both cases, so that the period of two years seems to be critical in the sense of monitoring the return of a state to itself. In the case of inefficiency it represents a sort of “safe mean time span” for managers to try to change the operating conditions facing inefficient units. Since the operation plans already point to “optimal changes” by unit, managers may evaluate for which units those changes would be feasible within (the next) two years. Note that on average an inefficient unit will return to inefficiency four months before it may reach efficiency for the first time, if no managerial action is taken.
5 Summary and conclusion
Upon assuming the Efficiency Principle as a guideline to organizational evaluation, this paper presented a model for library assessment in the short and long runs by combining two approaches – the DEA approach to efficiency analysis and the Markovian assumption that introduces a long run perspective. From a methodological viewpoint it extends and improves upon previous work by Carvalho et al. (2012-a, b).

We proceeded in three steps. The first step is typical of DEA-based efficiency analysis since one or more DEA models (Estellita Lins and Angulo-Meza, 2000) are estimated to provide efficiency scores allowing (Marinho, 2001) to rank the sampled DMUs according to their relative efficiency. The second step is also typical in DEA applications and consists in identifying “optimal” quantitative (re)allocations that would signal to managers how they might, if so wanted, lead inefficient DMUs toward the efficient frontier. These quantities might equally help to evaluate allocative gaps between optimal prescriptions and observed allocations.

The third step introduces a very simple long run perspective. We assume that any DMU can be in either of two aggregate states – “efficient” or inefficient” – and that the set of DMUs is accordingly classified in either state according to the proportion of DMUs in each state. The Markovian assumption (Kemeny and Snell, 1972) of constant transition probabilities between states will then allow to establish a long run equilibrium distribution between states. Findings have shown that the three step model uncovers quantitative aspects that may be of assistance to managers committed to efficiency in the short and long runs.

In the first two steps, typical DEA models provide both rankings and operation plans that not only help evaluate library performance, but may also assist inefficient library units in their quest for efficiency.

In the third step we rely on Markov Chains for long run assessment. We first compute an aggregate measure of the distribution of the productive system (the “organization”) between two states – efficient or inefficient. To the extent that individual DMUs are assigned to a “state” according to their performance and that transitions refer precisely to those states, our approach is aggregate in the sense that only “systemic” information remains. Levels of efficiency scores relating to specific DMUs are in that sense voluntarily lost (cf WANG and HUANG, 2007, p. 1306). Nonetheless, the number of states does not imply any limitations in practical applications. At the moment of writing we are completing an application where three states are defined in order that classification inaccuracy may be depicted.

The other useful application of the Markovian approach provides better knowledge concerning the time delay required for efficiency to be attained for the first time when a prescribed operation plan happens to be adopted, as well as about the time during which an undesired (inefficient) situation will persist if that adoption is postponed. This timing aspect may help library managers in preparing their planning and control schedules and figures with a view toward the efficiency endeavour. For example, an inefficient unit will on average return to inefficiency four months before it attains efficiency for the first time, so that managerial attention to such time lags may become critical.

Future research - based on alternative ways of using scores to define “states” and on alternative ways of obtaining a transition matrix to start the process - is likely to provide
better theoretical as well as empirical information that will allow for a better assessment of the proposed model. Some alternatives might be a simply “statistical” treatment (e.g., “above the mean” as in Wang and Huang, 2007, p. 1307) of what “good state” means or else the use of fuzzy concepts to help define that same idea of “good performance”. One remaining though important issue refers to how to deal with errors in measuring efficiency scores. Econometric modeling will likely be needed to explore and understand the effects of (statistical) errors.

Last but not least, the long run is here depicted in a very simple way and the “short memory” assumption involved in Markovian approaches may appear inappropriate in many contexts. The adequate approach to this issue still requires more work.

References


